MATHEMATICS TEACHING

THE BULLETIN OF THE

ASSOCIATION for TEACHING AIDS IN MATHEMATICS

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No.1 —NOVEMBER 1955 3/-

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ASSOCIATION FOR TEACHING AIDS IN MATHEMATICS

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This Bulletin appears twice yearly.

EDITORIAL

For the first time we are to appear in print! At the time of writing we can only guess what we are going to look like, and we live dreaming of printer's ink and correction signs; but by launching this journal we hope to provide a fuller and better service to our members, and to reach a larger circle of readers outside the Association. It will be the policy of the Editorial Board to present articles and news items covering the whole field of mathematics teaching, paying particular attention to the use and development of teaching apparatus and visual aids. We believe that there is need for a periodical devoted to these aspects of mathematics, and it is not our intention to imitate the type of material to be found in other journals, but to develop along lines of our own. Our task is to supplement the work being done elsewhere and not to compete with it.

The teaching of mathematics requires constant research; and research which aims to advance knowledge of the craft of teaching is just as difficult as research which aims to advance knowledge of mathematical techniques, and perhaps it is even more important. No one can do it better than those who are actively working in the classroom, and this journal is a means by which practical classroom experience can be passed on to others.

We aim to cover the whole field of teaching—nothing is too elementary and nothing is too advanced. We await your contributions.

Later in this number is the second article by Mr. R. D. Knight about the differential analyser which he built at his school. We live in an age of automation, and computors, if put to wise use, can arouse great interest in mechanically minded boys. The government surplus shops are selling bombsight computors at an absurdly low price. Can anyone tell us how to make a demonstration model out of one?

MATHEMATICS TEACHING AND THE A.T.A.M.

Our Association was conceived four years ago, in November 1951, at a week-end conference in Oxfordshire on the teaching of mathematics. It became, in a few months, a very promising body of active people ready to use their enthusiasm and skill for the promotion of something much wider than their own personal field of interest. In fact, the teaching of mathematics at all levels, through teachers with very varying mathematical backgrounds, was clearly in the forefront of the minds of those members who actively supported the work of the Association. It was not this or that gadget that mattered; one's own hobby did not interfere with the main task. It was accepted that the value of teaching aids could not be gauged outside classroom situations, and the means by which the efforts of the Association could be integrated with the teaching in schools became more and more the centre of the Committee's "social" projects. Now, after four years of life, we can say without hesitation that the Association has proved its use to teachers in both primary and secondary schools. How has this been achieved?

The mainspring has been the maintenance throughout of relations with schools and contact with the problems in the reality of a self-educating process. All those who joined the Association did so either because they had already worked with teaching aids on one or other of the problems in questions, or were in need of help in acquiring more technical knowledge for producing or using aids, or because they wished to support the activity of a group engaged in discovering what was needed for the improvement of the teaching of mathematics. The Association was therefore a somewhat heterogeneous body, needing to educate itself to a better understanding of how to develop its own work successfully while at the same time making a significant contribution to the solution of the problems met. In spite of the risks involved, the principle was accepted that the test of the value of teaching aids was the demonstration in the classroom in the presence of critical teachers, and lectures and purely verbal presentation came more and more to be replaced by demonstrations offering concrete instances for explanation, criticism and discussion.

In this way, the Association avoided the danger of becoming over-specialised and of making claims that could not be substantiated. It was the teachers themselves who decided what use *they* could make of the aids in their lessons and who gave ready support to the ones that proved successful.

Moreover, from the start it was the policy of the Committee to take the initiative instead of waiting for teachers to become members of the Association and thus learn of its work. In the first eighteen months, three week-end seminars were organised in Oxfordshire and a one-day exhibitions with demonstrations in London. During the next eighteen months the Association, in conjunction with other educational bodies (Institutes of Education, N.U.T.), arranged one-day exhibitions with demonstrations in Manchester, Exeter, Birmingham and Doncaster, and held two week-end seminars, in Birmingham and Stockport. The programme for the remainder of 1955 and the 1956 session includes meetings at Southampton, an open day in London and two week-end seminars in Bristol and the North of England. Fuller details will be circulated later. When it has not been possible to organise large meetings, individual members of the Committee have been invited to speak on behalf of the Association to groups of teachers.

From what has been said it will be clear that the Committee continues to be concerned with the relation of the work of the Association to the profession as a whole. But the technical advances which are the result of the individual work of its members must not be forgotten. A most promising film unit has been formed and will welcome any members interested in that aspect of the work. New films are being made, the problems involved in this art are being studied, and filmstrips and other aids suggested by members are being produced.

The Association is in constant contact with many other groups working on similar lines in the United Kingdom and abroad. Many of its members are also members of the Mathematical Association, some are members of the International Commission for the Study and Improvement of the Teaching of Mathematics, and the A.T.A.M. exchanges this Bulletin for their magazines and journals, which can be borrowed by members from the Librarian.

For information, we give below the titles of the A.T.A.M. seminars for the years 1952-56:

1. The evaluation of aids in teaching mathematics. (Ipsden.)
2. The rôle of intuition in mathematics learning. (Ipsden.)
3. The techniques of models and film and filmstrip making. (Ipsden.)
4. Research in teaching aids in mathematics. (Birmingham.)
5. Teaching mathematics in Modern Schools: The learner-centred approach. (Stockport.)
6. Knowing and proving in mathematics. (Bristol.)
7. Experimental and intuitive mathematics. (North of England.)

It is hoped that with the assistance of all members of the Association it will be possible to undertake an investigation which will assist the profession in taking its bearings with respect to the activity of its members. We can, for instance, study the comparative value of the aids suggested for the teaching of fractions, of areas and of loci. A more detailed programme of this proposed activity will be submitted in a future journal.

An Association with so live a field of study cannot fail to find ever new and interesting subjects for study, and this journal can become a useful intermediary and serve as messenger to the profession.

THE DIRECTOR OF STUDIES.

STATEMENT OF ACCOUNTS

January 1st to October 21st, 1955

INCOME

	£	s.	d.
Balance brought forward, January 1st	20	5	1½
Subscriptions	50	18	4
Commission on Sales ...	3	6	8
	£74	10	1½

EXPENDITURE

	£	s.	d.
Postage (Treasurer)		9	1
Stationery, etc. (Treasurer) ...		5	9½
Assistant Secretary, for Postage of Bulletins, Stationery, etc.	6	0	0
Cost of Duplicating Bulletins	24	5	6
Expenses of Birmingham Week-end, February 11th to 13th	3	10	0
Cost of half Return Fare for Member to attend London Committee Meeting ...	1	11	5
Balance in Hand	38	8	4
	£74	10	1½

B. Briggs, *Hon. Treasurer.*

A MODEL DIFFERENTIAL ANALYSER: II

R. D. KNIGHT

(A previous article described the construction of a school differential analyser. The author here discusses some teaching which has been done with the aid of the machine.)

THE use of the machine in Sixth Form work on differential equations has proved to be of value in that it stimulates the imagination.

Diagram I shows the set-up for the integration of $y'' = -my$.

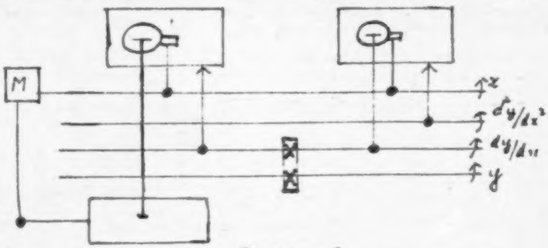

Diagram I

The best way to proceed has been found to be to set up the machine without informing the class what equation has been inserted, let them see the sine curve drawn on the output table and, after a close study of the machine, set them to discover the equation and interpret the resulting curve. The behaviour of the oscillating pencil gives the class a visual demonstration of Simple Harmonic Motion and the form of the equation leads to the definition. The integral in the form $y' = \sqrt{m(a^2 - y^2)}$ is produced if y' is plotted against y, the solution being a circle if $m = 1$.

By using an adding device to add together the rotations of the first and second derivative shafts it is possible to study damping effects; and by using a graph to represent, say, $b \cos px$ on the input table solutions of $y'' + ky' + mx = b \cos px$ are produced.

These give an interesting visual demonstration of the phenomenon of Resonance. No one who has seen the pencil on the output table begin to take flight towards the infinite can doubt the existence of what is otherwise, for the young mathematician at any rate, a rather theoretical point in the solution of the equation. Moreover, he can make an empirical investigation of the relation between the frequencies of the two parts of the equation when resonance occurs.

CORRESPONDENCES

As there is an obvious (1,1) relationship between points on the input plane and points on the output plane once the machine has been set up, I have been led to use it for purposes quite different from those for which it was designed.

(*a*) *Orthogonal Projection*.—By using a single integrating table as a variable gear for multiplying by a constant fraction the machine can be used to demonstrate Orthogonal Projection. The set-up is shown in Diagram II.

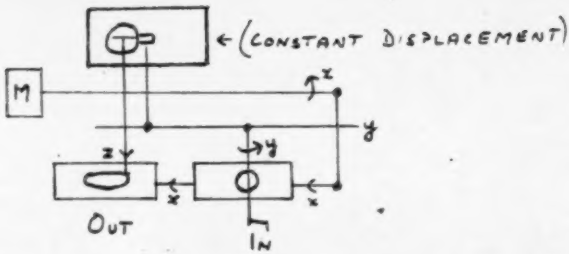

Diagram II

The axioms of the geometry are here replaced by the "machine set-up," i.e. by definitions inherent in the machine. All the theorems of orthogonal projection can quite easily be "proved" and with a little mechanical ingenuity a circle projected into an ellipse and vice versa.

(*b*) *The Quadratic Equation.*—The machine set-up for this study is a little more complicated and involves the use of two adders. The diagram is as follows:

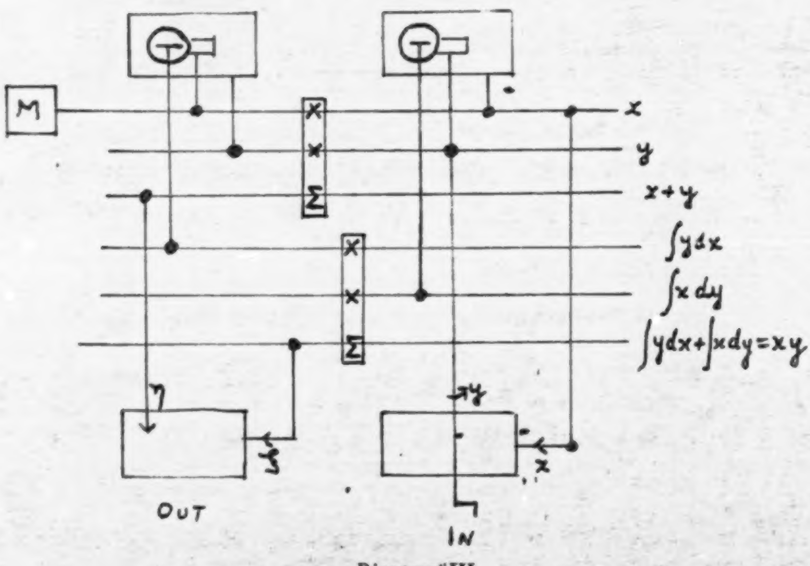

Diagram III

The relationship is between points (x,y) on the input table and points (ξ,η) on the output table where $\eta = x + y$ and $\xi = xy$. Thus x and y are roots of the equation $t^2 - \eta t + \xi = 0$.

The machine enables us to draw the relation connecting the coefficients in the equation when a certain relationship between the roots is specified. For example, draw the graph of $x = y$ on the input table. When this is followed the machine will draw $\eta^2 = 4\xi$ on the output table, thus giving the condition for equal roots. There are other fairly obvious extensions.

RESEARCH PROJECTS

It is useful, but not always easy, to find suitable topics for Sixth Form research. Work on the following ideas looks promising:

(i) Referring to Diagram III, the straight line $\xi + p\eta + q = 0$, which is defined when p and q are known, corresponds to a curve $xy + p(x + y) + q = 0$ on the input table. Thus there is a (1,1) correspondence between a point on a given line in the ξ,η plane and a pair of an involution. An immediate consequence is that the double points of the involution correspond to the points where $\xi + p\eta + q = 0$ meets $\eta^2 = 4\xi$. Thus there are either two real double points, two coincident ones (whatever that may mean can be investigated) or (since we are concerned with the real plane) none at all. This opens up interesting fields.

(ii) Production of stereoscopic drawings for three-dimensional curves. See *Mathematical Gazette*, Vol. XXXIV, p. 276.

(iii) The machine can be used to produce a graphical solution of equation which cannot be solved by elementary methods. The most obvious, $y'' = -m \sin x$ has been done by a Sixth Form boy and his paper on it accepted for publication in the *Mathematical Gazette*.

Others we shall try to do are:

$$y'' + py' - q \sin y = 0$$
$$y'' + q \sin y = P \cos kx$$
$$y'' - k(1 - y^2)y' + y = 0$$

* * *

THE mathematician is a man who, in his highest flights of imagination, is familiar with realities so augustly remote from the daily round of human life, and is the master of a craft as inaccessible as painting or sculpture, so that he must regard himself, to some extent, as privileged among men as is the artist. Also, all those occupied in the most abstruse mathematical fields are bound to experience a sensation of electness, and can hardly escape twinges of superiority—and with much more justification than any but the greatest artist.

Wyndham Lewis: *The Demon of Progress in the Arts.*

ABSTRACT MODELS III
SOME MODELS OF FUNDAMENTAL IMPORTANCE

T. J. FLETCHER

My dictionary defines a model as "a representation of structure." So whilst a model is usually thought of as being a concrete object, it may well be abstract—because one abstract system may represent the structure of another. The finite number system with only five elements in the first article of this series is important not merely as a toy, but because its properties illustrate certain important features of other number systems, even when they are much more complicated.

In a similar way I have explained elsewhere[1] how a model geometry based on this number system, and consisting of only 25 points, may be used to illustrate many of the properties of ordinary geometry from the most elementary stages up to projective geometry, the Argand diagram and beyond. Models of this type can often be used with profit in elementary teaching, and to overcome any suggestion that they are mere trivialities and to emphasise their fundamental importance we will now consider two papers by E. V. Huntingdon[2] which appeared in America in 1905. Although they are now fifty years old, they remain a *tour de force* of this type of abstract model construction. The first is concerned with giving a set of postulates for Real Algebra and the second with giving a similar set for Complex Algebra.

Studies of this kind reveal the assumptions on which ordinary everyday mathematics depends, show what is fundamental and what is only secondary, and, furthermore, indicate the type of mathematics which arises if different assumptions hold.

One remarkable thing that these two papers bring out is the comparatively large number of postulates on which ordinary algebra is based. One cannot say that algebra is founded on any particular number of postulates as there are alternative sets which will do as well as one another; and also the number necessary depends on such questions as whether multiplication is to be defined in terms of addition or whether it is to be regarded as a separate notion involving axioms of its own. Huntingdon seeks to give sets of postulates conforming as closely as possible to familiar forms of presentation, and he gives a set of 16 which apply when multiplication is defined in terms of addition, and a set of 20 which apply when it is taken as a separate idea. (He also gives some alternative sets with different numbers.)

All of these postulates really are necessary, because without any one of them the definition of the system would be incomplete—the system would be noticeably different from the algebra which it is the purpose of the postulates to define. It may be wondered how one could demonstrate this; and it is precisely at this point that Huntingdon constructs models with such skill.

Real Algebra is based on the notions of a *class of elements* connected by the two fundamental relations of *less than* and *plus*. The 16 postulates mentioned above suffice to define all the other notions of algebra in terms of these three. To show the mutual independence of these postulates 16 models are constructed, each using

some class of elements and an interpretation of *less than* and *plus* such that the model does not conform to one of the postulates but does conform to the remaining 15! The writer thus demonstrates that each postulate in turn is truly independent and is not derivable from the others; for, as he says, "if it were every system which had the other 15 properties would have this property also, which is not the case."

A few examples will indicate the general idea. A finite arithmetic (such as the one in the first article of this series) satisfies all the postulates except one of those about *less than*. In a finite arithmetic it is not possible to interpret *less than* in such a way that if a is less than b and b is less than c than always a is less than c. These finite arithmetics display rules of addition (and multiplication) of the same form as ordinary arithmetic but they are ordered differently.

In another model a perfectly self-consistent algebra arises by interpreting "a plus b" to mean "b" always. This is an example of a system that does not satisfy the postulate that "a plus b" equals "b plus a." That is to say, addition in the system is not commutative. It is, however, associative, that is, $(a + b) + c = a + (b + c)$.

As an example of a system in which addition is commutative but not associative we are given one where "a plus b" is interpreted as $2(a + b)$ (in ordinary notation), when both a and b are positive or both negative, and as $a + b$ in the normal sense when they are of opposite sign. This example is harder than the previous one, but regarded as a model it is in fact more interesting, because in the second model there is only one element (zero) with the property $z + z = z$, whereas in the previous model every element has this property. Now it turns out that systems in which there is a unique zero element are more interesting than those in which there is not, chiefly because some of the postulates are so worded that they are conditional on there being a unique zero in the system. This second model has more working parts!

When multiplication is defined independently of addition, Huntingdon requires 20 postulates. Among the models illustrating these is one in which "plus times plus" is "minus." "Minus times minus" is then "minus" as well, and whilst there are many schoolboys who prefer to use this rule they would almost certainly regard "plus times plus is minus" as a high price to pay for it!

The second of these two papers proceeds in a similar manner to give a set of 28 postulates for Complex Algebra, and to prove them independent by constructing 28 models. In this paper multiplication is taken as a notion independent of addition and the postulates are framed accordingly.

Among the unusual systems of mathematics which are discussed we may note one in which "a plus b" is interpreted in the usual way, but in which "a times b" is interpreted as $a + b + 1$. This system obeys all the 28 postulates except the one that $a(b + c) = ab + ac$.

This paper also gives a system in which "a plus b" is interpreted as $ab(a + b)$, except that it is $a + b$ (in the normal sense) when a or b or $a + b$ is zero. This system is remarkable as it obeys *all* the postulates, and so from the point of view of

formal manipulation of symbols it is indistinguishable from the algebra which we use every day.

One may ask where all this leads, and question its relevance to elementary teaching. Is it merely further evidence of the lengths to which highbrow mathematicians go when they are constructing playthings for their own amusement? I think not. Each of Huntingdon's models was constructed with a definite purpose in mind, and even if the purposes are not our own immediate concern they show how models have a part to play at all levels of mathematical exposition. The really important thing is that models should display ideas and provoke thought.

REFERENCES

1. T. J. Fletcher, Finite Geometry by Co-ordinate Methods, *Mathematical Gazette*, February, 1953, No. 319.
2. E. V. Huntingdon, *Trans. Amer. Math. Soc.*, Vol. VI, 1905.

DONCASTER

A Refresher Course on "The Teaching of Mathematics" was organised by the Doncaster and Don Valley Associations of the National Union of Teachers, in collaboration with A.T.A.M., on Saturday, October 8th, 1955.

After a felicitous opening by Councillor Mrs. Cover, this one-day course swung into full and varied activity. There were demonstrations and lectures going on for most of the day, often in two places at once; and besides these, a large and varied set of exhibitions, ranging from Nursery to Sixth Form work, which drew constantly moving crowds.

At the lectures, the enthusiastic appreciation of the audiences was unmistakable. Of the 350 teachers present, about 230 were interested chiefly in Primary work; so that great waves of people flowed into the hall for each of the demonstration lessons given by Dr. Gattegno, and again later for the lecture given by Miss Thyra Smith. As usual, the Gattegno demonstration lessons (with 6-year-olds, and later with 9-year-olds) provoked strong reactions, not all favourable; and some of his hearers, no doubt, will never summon the energy or perception needed to study the Cuisenaire methods closely. Many others, however, seeing this approach for the first time, were impressed sufficiently to want to enquire further. To the children, as always, his lessons brought a joyous glimpse of uncluttered truths.

Miss Smith's lecture on "Arithmetic in the Primary School" was erudite and lucid; and, by the nature of its content, was easier of access into the average teacher's thought. She included a vivid and interesting account of some exploratory work done on measuring by a class of Primary school children; and some of the results of this work, the children's own efforts, were on view in the Primary Exhibition room.

In the Secondary field, Mr. Collins astonished a set of 14-year-old boys, and some of his adult hearers too, by his masterly filmstrip lesson on the graphs of quadratic functions. And throughout the day Mr. Harris received a steady trickle of interested enquirers, in the room devoted to the demonstration of films and

filmstrips. Mr. Fletcher showed one of his own films, and some of Nicolet's, to a crowd of people whom they deeply impressed, and explained to them his views as to how their use can enrich and extend the study of geometry.

Miss Giuseppi's lecture was concerned chiefly with the teaching of Secondary Modern girls; and her audience, mostly women, were genuinely grateful for the wealth and fertility of ideas which she put before them.

The last lecture of the day was Mr. Fielding's, in which he explained the procedure of the mathematics course he runs for his Secondary school pupils, which is based on practical work. He showed some of the excellent teaching apparatus which he uses; and the mixture of inventiveness and ordered planning which he revealed added yet further richness to the day's experience.

Such a vivid series of experiences needs a framework of introduction and summing-up, as every teacher knows; and these were well provided by Mr. Munday, H.M.I., and others, at meetings of the whole conference chaired by Mr. Wynne, the President of the local branch of the N.U.T. The workers of this branch, the Doncaster and Don Valley Associations, are greatly to be congratulated on the highly successful organisation and happy atmosphere which characterised the whole day. Much credit is also due to Mr. Adams and his helpers who produced the various exhibitions; as with the lectures themselves, there was nothing shoddy here, nothing insincere, and much that was enterprising, imaginative and delightful.

Finally we ask, what comes next? It certainly seems that such courses as these have a message to bring and an impact to make, and that many more will be needed before we reach saturation point. Perhaps it may soon be possible to include opportunities for group discussion, after some of the lectures and demonstrations —there was little chance of direct participation by the audiences during the Doncaster day. All suggestions must, however, be tentative—it is impossible to reduce the question to one of direct proportion, otherwise we should pose it in the following form, and know the answer at once: If one A.T.A.M. for one day in one school influences 300 teachers, in how many schools must the A.T.A.M. be to provoke all mathematics teachers all the time?

M. J. MEETHAM.

* * *

I ADDRESS myself to teachers of mathematics of all grades and say: *Let us teach guessing!*

I do not say that we should neglect proving. On the contrary, we should teach both proving and guessing, both kinds of reasoning, demonstrative and plausible. More valid than any particular mathematical fact or trick, theorem or technique, is for the student to learn two things:

First, to distinguish a valid demonstration from an invalid attempt, a proof from a guess.

Second, to distinguish a more reasonable guess from a less reasonable guess.

G. Polya: *Mathematics and Plausible Reasoning.*

LYME HALL SEMINAR—"THE TEACHING OF MATHEMATICS IN MODERN SCHOOLS"

HAVING braved the deer-haunted, mile long drive by night, the draughts of pneumonia corner and the almost overpowering splendour of the pile of Lyme Hall, thirty-six people complacently attended on whatever awaited them. With Confession at eight, came disillusion. Each explained his interest in mathematics; a long, short, or varied experience and an expectation of a new approach to maths enriched by tips and teaching devices. But they had not met Gattegno! Within minutes of having wagged their tails so nicely they were told they were not to get tips but to give their experience; to meet a challenge, not to acquire parts of a guide book.

The week-end task was clearly defined:

(i) What is mathematics as a mental activity?

(ii) In what way can the teacher help the learner in learning certain fields of knowledge?

Attention must be focused on the learner. The syllabus must be "learner-centred" (there are no types of children), and we must determine the guiding principles. Thus far Gattegno; now it is our turn. First one, then another; would nothing please him? One was "projecting his anxieties," another was met with silence, yet another was shifting from the problem that concerned him, and periodically a pearl was dropped from the Leader's lips. It was all very distressing to the listeners. In language there were words, in mathematics there were not words; activity in a situation produces mathematics as we learn to appreciate that situation; we must watch ourselves learn so that we might know what it is to learn. Inevitably the theory of sets came on the scene! The laws of mistakes were based on isomorphisms. We do not see what the children do. We know there is an automatic process. In this there is a shift from one awareness to another according to certain laws. This is Mathematics. It was 11 p.m.

The following morning clearly bore the mark of the night before. We wandered through the gardens and the park. This was not what they had come for I was told, "How am I going to use this in my classroom?" "Where do we go from here?" But the leaven *was* beginning to work. "Perhaps I am wrong . . .? Perhaps I might do something else . . .?" So with an air of resignation, yet also with an expectancy of some challenge, having dealt with the learner the Seminar began its attack on Method before syllabus, and here we saw the method of the leader. They had come for bread and he gave them a stone. Now the stone was seen to glitter. First came belief in the Faith:

(*a*) Everyone is capable of creative mathematics.

(*b*) People are able to alter attitudes and get methods changed.

Through fractions we tilted at tradition. Again and again the brash young men gave of their experience, and invariably they found it was merely an attitude, an emotion, an anxiety or some projection (but not mathematical!). Gradually they came to respect the rock of faith: "If tradition is not right, then we can start a change. We must move on to other methods centred on the learner." A crusade? A revolution? Tradition overboard? "Emotion, habits and liking for habits, are these

those to which you cling? Is revolution possible?" We looked for the revolution and our stone seemed to acquire a polish: the revolution in attitudes or minds comes not by providing a doctrine but by methods of work without upsetting prejudice. Here followed the sermon.

The teacher goes into the classroom not to teach but to learn: he always learns something new about the topic in hand. Through Cuisenaire material we wandered until we saw that the child finds things immediately which we are not prepared to find. Our search was now for material and not for method. Every teacher is entitled to the same freedom as the child. Through materials "New concepts do not resist the old concepts. Many things consist in taking steps, etc., which even for mathematicians are uncertain. Therefore allow the child to make uncertain steps until the certainty appears; not present the final decided certainty. Logical proof comes from the conviction of the validity of what we are trying to prove. A mathematical statement is the abstraction found from experience which may or may not be true."

Now the tone has changed. The stone takes on the qualities of the diamond. We have seen some materials and we have two pregnant ideas. And we reflected on them as we toured Lyme Hall after lunch. We looked at the splendour of past ages. Here slept James I. This is the cloak which Charles I wore on the scaffold; here is the best in furniture and pictures of a world we have seen through a cinema screen: this was Lyme Hall. We stepped back into the Seminar.

The meeting bubbled in anticipation of Gattegno's Lesson. Unfortunately there were no children, only members of the Seminar. Yes? Of course—the set! But this time an elephant; somehow a quotient elephant, it might be an equivalent sub-elephant, but always an elephant—what's in a name? It is always a set. The demonstration was marred somewhat by students who weren't really pupils and by a teacher who forsook an avowed intention in pursuit of a new experience. "The most formal of formal lessons I have ever seen," declared one victim—in his blindness he had forgotten his own activity.

By this time most members of the Seminar were mentally battered, a state of mind which films, French songs and Spanish love ballads did much to repair, but most people were glad to seek their beds.

The final discussion got off to a bad start. "A learner-centred syllabus" found us once more, "anxious, attitude-ridden and projective" (but not mathematical)! Some one mentioned "conditioning," and we knew that political anxiety had reared its ugly head. A purpose was served, however, for now came the application:

(*a*) We start with our pupils who know what they know, which we must find out experimentally.
(*b*) We provide a simple but pregnant situation (e.g. a geoboard).
(*c*) We make the children comprehend the situation.
(*d*) We allow them to explore the situation by action.
(*e*) As a consequence of a variety of findings recorded individually or given to the teacher we use the blackboard to polarise the observations round uniformities, order, etc.

It is enough to find something which can have resonance in a situation for

things written learnedly on a blackboard are essentially simple. Learning mathematics is learning an attitude of mind. With older children we use a complex situation which *they* can conveniently command, not one at the level of perception of the teacher.

Now came the tips (in spite of the first warning): shirt sizes, suits, photograph of a mosaic on a mosque, number squares, shapes and square inches, algebraic transformations, weights, the children in a class were pregnant situations. We were warned to beware of projecting our own interest, but we heeded not; we had something we could use, we had something we could imitate. The stone *was* precious.

Finally came the reciting of the Creed. Ten had departed, only twenty-six remained. Now we should know whether they accepted because they were too fatigued to continue resisting or because of conviction. We didn't really find out. (Do we ever?) All admitted to a change of some sort. A minority did not see how it would affect their teaching. Many felt that we ought not to have included "Secondary Modern School" in the title, but had we not done so, confessed they would not have come. A majority affirmed.

One final benediction: "Beware of a strong personality bludgeoning the children, of emotion. Go forth and change our class."

Behind the scenes, almost unnoticed but always efficient, kind, considerate, moved Mr. Birtwistle at everyone's command. We expressed our thanks to him for the way in which he had organised a memorable week-end—we shall remember him gratefully whenever we recall Lyme Hall.

The Seminar was over.

Looking back one sees everywhere Gattegno. Every discussion centred around him, present or not. Students spoke (but with affection) of the way in which he bludgeoned every contribution to discussion; they felt he was convinced that only he was right and could be right—"did he fall into the error of the 'unco' guid?" But of all things, they could remember Gattegno. Gattegno is Gattegno.

C. HOPE.

THE BRITISH ASSOCIATION IN BRISTOL, SEPTEMBER, 1955

R. M. FYFE

It was my first experience of attending a meeting of the British Association for the Advancement of Science. I was impressed by the first-rate organisation, and believe that visitors who were strangers to Bristol were also easily able to find their way from place to place.

Before the week began, there was the fun of examining the promised menu of meetings and excursions and deciding how much of what one would like to do could be fitted into the time. The fourteen main sections all had interesting lectures arranged for each morning session, at least half of which I should very much have liked to attend. On the Tuesday one "registered," received a very handsome book

on Bristol as a free gift, and collected tickets for the receptions and excursions for which one had booked.

True to my profession as a teacher of mathematics, I faithfully attended the meetings of that section, but must confess that I was disappointed. The fault may have been merely my own ignorance, but I felt that the level at which the lecturers aimed was a little off tone. The topics of Symbolic Logic, and Stochastic processes, in particular, which were new to me, seemed to be dealt with too rapidly and without concrete illustration, so that the beginner was lost after the first few moments, whereas I doubt whether anyone who already had some knowledge in these fields would have learnt much that he did not know. I suppose mathematics is the most difficult section for which to cater.

The Education Section held some good meetings, which included a provocative lecture by Dr. Bronowski. There were joint sessions, with Mathematics and Physics on Monday, and with Chemistry on Tuesday, at which the training of the young scientist was thrashed out. These were well attended. Another joint session which was very popular was on the subject of heredity and DNA. Prof. Haldane, as one of the speakers, gave an amusing and highly informative lecture.

My greatest enjoyment came from the showing of scientific films. They were widely varied and the programmes very well selected. It was a real privilege to see the film *90° South* of Scott's 1911 expedition, made by Herbert Ponting, and also the *Everest* (*1922*) film, which was explained to us by Captain Noel himself. The German research films of a spider and of hamsters were delightful. The mathematical films were well selected, and were enjoyed by a large audience.

Another highlight of the week was a visit to the production sheds of the Bristol Aeroplane Company, where we were most courteously welcomed and shown all the stages in the making of jet aircraft from their beginnings as wax mouldings to the final stages on the test beds.

Excursions were arranged for every afternoon, with whole-day outings on the Saturday and Sunday. Everyone I spoke to enjoyed the excursions, and mine included an enthralling examination of the beautiful city of Bath; and an intimate walk round Bristol with the City Archivist, Miss Ralph, who finished our tour by taking us to the inner sanctuary of the Council House to see the very fine City Regalia.

Perhaps, next time, I shall attach myself to a completely different section; I throw this out as a suggestion to others who might also prefer to widen their horizons to allied, or completely diverse, branches of Science. As a matter of fact, much pleasure and benefit could be derived even if one attended no section meetings at all, as there were many lectures of general interest, the inaugural meeting and several receptions. The exhibition of Science work in the schools was very well prepared and I should have liked to spare more time to enjoying and examining it.

There certainly need have been no dull moments with so many and varied activities planned, and I do recommend all who have the opportunity to go to these annual meetings whenever distance makes it possible. Next year those who live in or near Sheffield will have the opportunity that I had this year.

FILMS

Base Line Measurement. 16 mm., sound, 12 minutes.

This film shows how the difficulty of obtaining an accurate Base Line for a primary survey of Great Britain was overcome.

Triangulation, the method employed, requires the accurate measurement of angle—a geodetic theodolite gives this to 0.5 seconds of arc, or approximately the angle subtended by a halfpenny at six miles—and the accurate measurement of length, in this case within one part in five hundred thousand, or better than three-quarters of an inch in six miles.

The fact that, for a triangle, three angles is not a case of congruency and that one side is required, is well done.

The equipment used, known as the "Macca" equipment (designed by the late Captain McCaw), makes use of a calibrated Invar steel tape suspended in a catenary under a known load. The film demonstrates this equipment in use and indicates the many sources of error, i.e. temperature, slope, variation in gravity, etc. Calculations are not given but provide a useful exercise.

The high degree of accuracy thus obtained is shown by the reproduction of a base line after 440 miles of triangulation, which agreed with direct measurement to one part in 97,000.

The Surveyor's Level, Part I. The Geodetic Level, Part II. 16 mm., sound, 23 minutes.

This film gives some of the theory and practice of levelling, with the emphasis upon the construction, checking and adjustment of the instruments. It illustrates, very well, how the theorist and practician have worked together to surmount their difficulties.

Part I.—The surveyor's level, used normally over short distances (about 3 chains), has an accuracy of one thousandth of a foot, much care being given to the spirit level and optical system to obtain this. The setting of the instrument is effected by a differential screw thread arrangement.

The remainder of this part demonstrates, with calculations, how to find and adjust the error due to the line of sight and true axis being out of alignment.

Part II.—The geodetic level, designed for use over much longer distances, has two main refinements: a symmetrical spirit level vial, which by rotation indicates the true level regardless of misalignment; and a parallel plate micrometer for the accurate estimation of readings between staff graduations.
the accurate estimation of readings between staff gradations.

The film concludes with a series of pictures showing the geodetic level in use along the Red River in Canada.

The two films contain well-selected material and a commentary suitable for Fifth and Sixth Forms, particularly for those following a course on surveying. The combination of live action and animation is good, but the speed of presentation is too fast. Full value can only be achieved by more than one showing. A booklet issued with the films helps to overcome this fault.

The films are offered for loan, and application for booking should be made to the Librarian, Cooke, Troughton & Simms Ltd., York.

IAN HARRIS.

* * *

THE CARDIOID. A mathematical film designed and directed by Trevor Fletcher. Produced by Sir John Cass College. Photography and technical advice by Polytechnic Films Ltd. 16 mm. or 35 mm., silent, 15 minutes.

When confronted by such a spectacle as the film *The Cardioid* presents, one has to take care not to be affected by the tremendous care and patience involved in the production. The technique which is being perfected is of a very high standard and there is no question of any criticism on that score. If we are to have mathematical films we must accept that the making of them involves a considerable volume of tedious repetitive work, but this must not affect any value judgments that we may make.

We should also be aware that knowledge of an author's (or in this case, producer's) intentions, as far as one can have knowledge of another's ideas, must have a considerable bearing on any criticism, constructive or otherwise, of his works. That Mr. Fletcher is aware of another aspect of this type of influence is apparent in the purposive absence of explanatory matter within the film itself. He states that this would create a "set" towards particular aspects only and, perhaps, prevent a deeper vision.

A third point to consider is to whom the film is addressed. One may first consider the general notion of any "presented" situation and state the distinction which can be drawn between those situation which "ask questions," or are dynamic, and those which are static, presenting a complete experience to the receiver. The writer feels that it should be the aim of the teacher to create a dynamic situation if genuine learning is to take place and although the other form is useful, it should never be used during the learning process.

In applying this general notion to the film situation we must be idealistic, for the concept of a dynamic (in the above sense) film situation implies ease of projection—that is, a classroom or lecture-room fitted with projectors for both moving films and "stills"—while the kind of arrangements that have to be made at present tend to help, by virtue of the "Pragnanz" notion from Gestalt psychology, the static completeness of the experience of a film.

Quite clearly, a film made with this "dynamic" viewpoint in mind would have to be linked with the personality of the teacher if it were at all elaborate. For the present one can only suggest a number of very simple situations in filmic form, lines crossing, circles intersecting and touching, etc., after the style of Nicolet's films, which may be used in a variety of ways by teachers with differing ideas.

It is with these several aspects, very superficially stated, in mind that the writer approached *The Cardioid* as a film to be discussed.

Straightaway it will be seen in accordance with the above to belong more to the "static" than the "dynamic" teaching situation. That does not mean to say that there is no provocation to enquiry but that it provokes enquiry and thought in the same way that a piece of music or a well-produced play does.

The initiate needs this film rather than the learner (if anyone can ever stop being a learner), but its existence raises the questions of the place of the film in (*a*) mathematics and (*b*) the teaching of mathematics. The remarks previously made on the latter apply in this case, but the other aspect, that of the film in mathematics (and *The Cardioid* in particular), is of an entirely different nature. This is an example of the "static" aspect of the "presenting" situation. Mr. Fletcher emphasises the dynamic (a different use of the word) nature of the moving diagram when he points out "that many questions on epicycloids can be solved mentally when they would be quite difficult if approached by a more static method." One may ask whether this is a very different statement from that which maintains that a problem in solid geometry is more easily solved by using bits of wire and thread to form a three-dimensional diagram.

If we are using the medium in this way, is there not too much in the film and should we not divide it into sections? It may be argued that there is more to it than that. The mathematical notions themselves, involving a new time vector, can be examined as such and so merely take this film as a primary datum for a new logical construct.

The questions are many and varied, and one thing is certain: the topic of the place of films in mathematics can be tackled from so many different (and opposing) points of view that every question asked must be answered according to the situation which led to the forming of that question.

If we were now to discuss whether, in this particular film, too long was spent here or too short a time there, or whether this could be cut or that could be emphasised, we would be talking each time from one of several points of view. It has been the intention of the writer to suggest the existence of these latter and also to suggest how easy it is to slip from one to another without noticing. It seems that Mr. Fletcher sometimes is thinking of the teaching situation and sometimes of the mathematics. Possibly he feels that as it is an expensive and arduous business it would be well to attempt to make a film which satisfies as many criteria as possible.

Finally, one can say that seeing the film is an exciting experience. The stimulus is mathematical, and the personal view of the writer is that it succeeds more as a piece of mathematics than as an aid to thinking about any static geometrical problem.

W. M. BROOKES.

FILMS ABROAD

NEWS has been received of mathematical film-making abroad. *Science and Film* reports a film entitled *Non-Euclidean Movements*, which was made for the Zentralinstitut für Lehrmittel in East Berlin, under the direction of Kurt Schröder. No copy seems to have reached England yet, but to judge by the synopsis this film deals with the most advanced topic to be attempted in the medium so far. Animated diagrams show the motion of the hyperbolic plane when a fixed point of the movement lies outside, inside and on the fundamental conic.

The first mathematical films to be made in Jugoslavia have recently been produced by Milan Krajnovic. They deal with the Theorem of Pythagoras,

Geometrical Loci, and Proportionality. Dr. Krajnovic has worked independently, and has not seen any of the mathematical films which have been made previously elswhere. The film on the Theorem of Pythagoras gives an animation of Euclid's proof (in a slightly modified form) and then extends it to provide a proof of the cosine formula.

May we hope to see these films in England shortly?

BOOK REVIEW

PRELUDE TO MATHEMATICS, by W. W. Sawyer. Pelican Books. Pp. 214. Price 2s. 6d.

Mathematicians already have good cause to be indebted to the publishers of Pelican Books, and now the appearance of a new book by the author of *Mathematician's Delight* increases the debt still further. This book has been written "for the person who is interested in getting inside the mind of a mathematician," and the publishers' introduction tells us that "the emphasis is not on those branches of mathematics which have great practical utility, but on those which are exciting in themselves." This book is written from a more mature point of view than its predecessor, and its subject matter is more advanced.

Of the many writers on mathematics, nearly all are concerned with tactical manipulation and very few consider the broad outlines of mathematical strategy. Dr. Sawyer attempts this more difficult task in the first five chapters, and discusses the qualities which make a mathematician—the ideas of generalisation, unification and the recognition of pattern. The remaining chapters discuss among other things non-Euclidean geometry, group theory, finite fields, determinants and matrices. The treatment throughout assumes no previous knowledge of the topics, and if it is possible to present them in a way which is comprehensible to the general reader then this book should succeed in doing so.

Every teacher will have his own opinions on sections here and there which might have been better omitted—is it worth introducing half a page on quarternions in a book of this nature? And how well will the concluding discussion of the group of an equation make the notion clear to those who know nothing about it to begin with? But these are small details and they are far outweighed by the many excellent sections which the book contains. One must have high praise for the treatment of projective geometry and its relation to perspective drawing. Many pupils' initial difficulties might be overcome if this practical treatment, which is strictly in accord with the historical origins of the subject, was more common. Seen from this point of view, many aspects of projective geometry are at once apparent, and the line at infinity is explained and handled in a most natural manner. (Incidentally, can anyone devise an equally natural method of explaining the circular points?)

The introduction to groups makes it quite clear that they can be taught at a comparatively elementary level in a concrete manner; and that there is no reason whatever why the elements should not be taught in the Sixth Form or earlier. This would be far more in the spirit of contemporary mathematics than, say, some of the geometry taught, and would have a vastly wider field of application. The idea

of a Galois Field is also explained in a simple manner, and furthermore a practical application is given! Few teachers could do this if asked off-hand.

It can be seen that this book raises serious questions for the teacher. A commercial publisher can produce a "popular" book at a price which is but a fraction of the price of a normal textbook these days, and large sections of the book deal with mathematics with which many schoolmasters are quite unfamiliar, and which they would certainly dismiss as being far too difficult to be taught in schools.

It is quite certain that mathematics teaching must inevitably contain a number of routine exercises in manipulation as training in technique, but how many things which are being taught at the moment have the elegant and stimulating attractiveness, and at the same time the logical simplicity, of many of the topics appearing here? The frontiers of mathematics are advancing at a staggering rate, and unless teachers make it a persistent aim to keep their teaching up to date the gap between the class and lecture-room and the front line can only become wider than it is already.

This book will help to bring about the attitude of mind necessary to bridge the gap. It can also be recommended to laymen who want to know a little of what mathematics is about, and above all to students standing at the beginning of a mathematical career.

T. J. F.

NOUGHTS AND CROSSES

ALL addicts of Noughts and Crosses will know that a properly played game on the usual 3×3 chart ends in a draw. Those who have experimented further will have found that if the game is played in three dimensions the first player can force a win on a $3 \times 3 \times 3$ chart, but that a draw results if both players play their best on a $4 \times 4 \times 4$ chart.

It is not difficult to play the game in spaces of more dimensions if charts are plotted side by side or on separate sheets of paper, and if obvious conventions are adopted concerning the definition of a "line." Continuing the train of thought suggested by the results of the simpler games, one is led to guess that a game of Noughts and Crosses played on an n^n chart in n dimensions should be a win for the first player, but a game played on an $(n+1)^n$ chart should be a draw.

I invite proof (or disproof) of this conjecture.

T. J. F.

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